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Stability of Compactification During Inflation

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Abstract

The possibility that inflation may trigger an instability in compactification of extra spatial dimensions is considered. In old, new, or extended inflation, the false vacuum energy results in a semiclassical instability in which the scalar field representing the radius of the extra dimensions may tunnel through a potential barrier leading to an expansion of the internal space. In chaotic inflation, if the initial value of the scalar field responsible for inflation is large enough, the internal space becomes classically unstable to ever increasing expansion. Restrictions on inflationary models necessary to keep the extra dimensions small are discussed.

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I. INTRODUCTION

If the fundamental theory of nature is a "higher-dimensional" one with extra spatial dimensions, it is necessary to hide the extra dimensions. The usual mechanism for hiding the extra dimensions is to assume that they form a compact internal space with a physical size small enough to have escaped detection. For currently available accelerator energies, this requires a size smaller than the Fermi length, or about 10^{-16}cm. This would not be surprising, since in almost all extra-dimensional theories the fundamental length scale is set by the Planck length, $l_{Pl} \equiv G_N^{1/2} = 1.616 \times 10^{-33}$cm. In the limit that the physical size of the internal space is smaller than the physical size of the external space, it is possible to dimensionally reduce the system (integrate over the extra dimensions) and obtain an "effective" $(3+1)$-dimensional theory.

The assumption that the extra dimensions form a compact space is quite reasonable since if the Universe is closed ($\Omega > 1$), the three observed spatial dimensions form a compact space (a 3-sphere, S^3). The remarkable thing is that there is such a disparity in the sizes—10^{-33}cm for the internal space and more than 10^{28}cm for the external space. Theories with extra spatial dimensions are many and varied. However all have common features of relevance for cosmology. In theories with extra dimensions the truly fundamental constants are the ones in the higher-dimensional theory. The constants that appear in the effective four-dimensional theory are the result of integration over the extra dimensions. If the volume of the extra dimensions would change, so would the "observed" constants. This implies that the internal dimensions must be static, or have changed very little since the time of primordial nucleosynthesis.[1]

The curious cosmology that emerges is one that has some dimensions large and expanding, and some dimensions small and static. Since expansion (or contraction) is the generic behavior expected, the challenge for cosmologists involves constructing models that have static extra dimensions. The basic approach is to assume that the higher dimensional theory is that of gravity plus a cosmological constant.[2] The extra dimensions are held static due to the interplay between the cosmological constant and either classical[3] or quantum[4] fields. Although the true mechanism in more complicated theories such as superstring models might be more complex, there must be some vacuum stress keeping the extra dimensions static and the toy models studied here may very well be relevant.

In the models that have been studied, the present ground state is stable against small fluctuations of the size of the internal space. Maeda[5] claimed that it is also stable against tunnelling under the potential barrier. In Section II we show that when other fields are introduced, the potential is changed in such a way that a semiclassical instability appears and there is a non-zero probability for the extra dimensions to tunnel out the potential keeping them small. On the other hand, the presence of scalar fields is required during the inflationary era so that their effect on the dynamics of a multidimensional Universe must be considered. In Section III we discuss the stability of internal space when old, new or extended inflation is considered. In this case the problem has a semiclassical nature: a calculation of transition rates is then performed. In Section IV the analysis is extended to Linde's model of chaotic inflation. Our results are summarized in a concluding section.

II. FROM N TO 4 DIMENSIONS

We will start with a theory of gravity in $N = D + 4$ dimensions with a cosmological constant $\bar{\Lambda}$ and some matter fields, for simplicity represented as a single scalar field $\bar{\psi}$. Upon dimensional reduction, the scalar field $\bar{\psi}$ will give rise to a 4-dimensional scalar field responsible for inflation (called the *inflaton*), and the degree of freedom corresponding to dilatations of the internal space will give rise to a second 4-dimensional scalar field known as the *dilaton*. The action is[6]

$$S = \int d^N x \sqrt{-\bar{g}} \left[-\frac{1}{16\pi \bar{G}} \bar{R} + 2\bar{\Lambda} + \bar{\mathcal{L}}(\bar{\psi}) + \cdots \right], \tag{1}$$

where $\bar{G}$ is the gravitational constant in $D+4$ dimensions, related to Newton's constant G_N by $\bar{G} = G_N V_D^0$ with V_D^0 the present volume of the internal space. The field $\bar{\psi}$ is assumed to appear as a minimally coupled scalar field:

$$\bar{\mathcal{L}}(\bar{\psi}) = \frac{1}{2} \bar{g}^{MN} \partial_M \bar{\psi} \partial_N \bar{\psi} - \bar{V}(\bar{\psi}). \tag{2}$$

Extra dimensions are assumed to be compactified to a D-sphere of radius b, whose present value is b_0. The metric then reads:

$$\bar{g}_{MN} = \mathrm{diag} \left[\hat{g}_{\mu\nu}(x)\,;\, b^2(x) h_{ij}(y) \right]. \tag{3}$$

After dimensional reduction, fields do not depend on the coordinates of the internal space (h_{ij} is just the metric of a D-sphere of unit radius), so that an integration over these coordinates yields only a numerical factor. Introducing the Newton constant G_N, the action (1) becomes:

$$\begin{aligned} S \;=\; & \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{1}{16\pi G_N} \left(\frac{b}{b_0}\right)^D \hat{R} - \left(\frac{b}{b_0}\right)^D \frac{D(D-1)}{16\pi G_N} \frac{\partial_\mu b \partial_\nu b}{b^2} g^{\mu\nu} \right. \\ & \left. + \left(\frac{b}{b_0}\right)^D \frac{D(D-1)}{b^2\, 16\pi G_N} + V_D^0 \left(\frac{b}{b_0}\right)^D \left[\hat{\mathcal{L}}(\hat{\psi}) - 2\bar{\Lambda} + \cdots \right] \right\}, \end{aligned} \tag{4}$$

where dots stand for other fields needed to obtain compactification. The ordinary Einstein–Hilbert action may be recovered after a conformal transformation of the 4-dimensional metric:

$$\hat{g}_{\mu\nu} = \exp(-D\sigma/\sigma_0)\, g_{\mu\nu}, \tag{5}$$

with the dilaton field defined by

$$\sigma = \sigma_0 \ln\left(\frac{b}{b_0}\right), \qquad \sigma_0 = \left[\frac{D(D+2)}{16\pi G_N}\right]^{1/2}; \tag{6}$$

it has the ordinary field dimensions of $(\text{length})^{-1}$. The desired final state is $\sigma = 0$, corresponding to $b = b_0$, and $\dot{\sigma} = 0$. This corresponds to a static internal space. The final 4-dimensional action is

$$\begin{aligned} S &= \int d^4x \sqrt{-g} \left[-\frac{1}{16\pi G_N} R + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U_1(\sigma) \right. \\ &\qquad \left. + \frac{1}{2} g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \exp(-D\sigma/\sigma_0)V(\psi) \right], \end{aligned} \tag{7}$$

where $U_1(\sigma)$ and $V(\psi)$ are specified below. In this last expression for the action, the metric tensor $g_{\mu\nu}$, and not $\hat{g}_{\mu\nu}$, appears; furthermore we introduced the field $\psi = (V_D^0)^{1/2}\hat{\psi}$ with canonical dimension $(\text{length})^{-1}$ [in the $D+4$-dimensional theory $\hat{\psi}$ has dimensions $(\text{length})^{-1-D/2}$]. In this conformal frame, the gravitational constant (the coefficient of the Ricci scalar) is constant, but the mass scale associated with the inflaton is not, due to the factor $\exp(-D\sigma/\sigma_0)$ in front of $V(\psi)$.

The potential $U_1(\sigma)$ of Eq.(7) contains contribution from (at least) three sources. The first source is the term in Eq.(4) proportional to $\bar{\Lambda}$. The second source is due to the curvature of the internal space, which appears in Eq.(4) as the term proportional to b^{D-2}. Finally there must be some other source to give a stable ground state. We will consider a general model that encompasses two compactification schemes, which we shall refer to as either *Casimir*, where an extra potential is given by the quantization of scalar fields in a compact space,[3,4] or *monopole*, where an extra vector field is considered for which the well known Freund-Rubin ansatz is taken.[7] Both cases are discussed in details in Ref. (8). The point is that the extra contribution is some (negative and D-dependent) power of the radius of the internal space; thus the curvature term can be balanced and a static solution $b = b_0$ (i.e., $\sigma = 0$) is allowed. Furthermore, this solution has non-zero energy, so that the N-dimensional constant $\bar{\Lambda}$ in the action (1) is tuned to ensure that an effective 4-dimensional cosmological constant does not appear. The potential $U_1(\sigma)$, shown in Fig. (1) for the Casimir case has the following expression:

$$U_1(\sigma) = \alpha\left[\frac{2}{D+2}e^{-2(D+2)\sigma/\sigma_0} + e^{-D\sigma/\sigma_0} - \frac{D+4}{D+2}e^{-(D+2)\sigma/\sigma_0}\right], \tag{8}$$

where $\alpha = (D-1)\sigma_0^2/b_0^2(D+4)$; in the monopole case it looks like very similar and has the same dynamical properties. When ψ is constant and has zero energy, the dilaton field is trapped at the minimum of this potential and is stable from the semiclassical point of view. On the other hand we must introduce a ψ field in order to have inflation. Thus, the evolution of σ will be governed by a potential of the general form:

$$U(\sigma,\psi) = \alpha\left[\frac{2}{D+2}e^{-2(D+2)\sigma/\sigma_0} + e^{-D\sigma/\sigma_0} - \frac{D+4}{D+2}e^{-(D+2)\sigma/\sigma_0}\right] + e^{-D\sigma/\sigma_0}V(\psi). \tag{9}$$

where $V(\psi)$ will be specified below for two different cases. In any inflationary scenario with phase transitions, ψ is initially in a false vacuum state. The potential is of the form:

$$V(\psi) = \lambda\left[\frac{1}{4}\psi^2(\psi-\psi_0)^2 - \frac{\epsilon}{2}\psi_0\psi^3\right] + \Lambda, \tag{10}$$

Here λ is the dimensionless ratio of the multidimensional constant $\bar{\lambda}$ [dimension $(\text{length})^D$] to the volume of the internal space V_D^0. The potential $V(\psi)$, shown in Fig. (2), has a true-vacuum at $\psi = \psi_T = \psi_0[3(1+\epsilon) + \sqrt{1+9\epsilon(2+\epsilon)}]/4$, and a false-vacuum state at $\psi = 0$. The constant Λ in Eq.(10) is now specified to be

$$\Lambda = -\lambda\left[\frac{1}{4}\psi_T^2(\psi_T-\psi_0)^2 - \frac{\epsilon}{2}\psi_0\psi_T^3\right] \tag{11}$$

in order to ensure that $V(\psi_T) = 0$. It will serve as the effective 4-dimensional cosmological constant to drive the de Sitter phase during inflation when $\psi \neq \psi_T$. For $\psi = 0$, the potential has the simple form $U(\sigma,0) = U_1(\sigma) + \Lambda\exp(-D\sigma/\sigma_0)$. The effect of the new term is to raise the energy of the minimum of the potential to a positive value, while leaving invariant the asymptotic behaviour for large σ. Since $U(\sigma=0,0) > U(\sigma=\infty,0) = 0$, the compactified vacuum is semiclassically unstable so long as $\psi \neq \psi_T$. There are two true ground states of the system. The first ground state is $\psi = \psi_T$ and $\sigma = 0$. This is the desired ground state corresponding to a compactified internal space. The other ground state is $\sigma = \infty$, for any ψ. This is the state to be avoided, corresponding to an expanding internal space. In the second case, the scalar field representing the radius of the extra dimensions tunnels through the barrier, and lowers the energy of the system by ever increasing expansion.

Compactification is not stable unless the inflationary stage ends before the internal space can grow. For this to occur, the inflaton must tunnel through the potential faster than the dilaton can tunnel through its own. The 4-dimensional appearance of the world is the result of a competition between the two scalar fields won by the inflaton. In the next section, we calculate the tunnelling rates in ψ and σ directions and show that the first one is larger than the second for reasonable choices of parameters, so that compactification of internal space is preserved in new or extended inflation.

We will also discuss stability in the context of Linde's chaotic inflation theory.[12] In this case the dynamics is totally classical, but the guidelines of the discussion are similar to the previous case. Here the introduction of the potential that drives inflation changes the dynamics of the dilaton field in such a way that for very large values of ψ, the barrier against evolution away from $\sigma = 0$ disappears, leaving the dilaton free to evolve classically during inflation. The potential assumed for chaotic inflation, Fig. (3), is of the form

$$V_C(\psi) = \frac{1}{4}\lambda\psi^4. \tag{12}$$

The relevant potential for chaotic inflation is $U_C(\sigma, \psi)$, which is Eq.(9) with $V(\psi) = V_C(\psi)$. Stability of compactification in chaotic inflation is discussed in Section IV.

III. SEMICLASSICAL STABILITY

Let us turn now to the evaluation of tunnelling rates in the two relevant alternative directions: toward an inner-space explosion, or toward the compactified vacuum. Only when the latter results to be much more likely than the former, will the process match the actual observations of an inflated, 4-dimensional Universe. The theory of vacuum decay in flat space is well known,[9] and in the thin-wall approximation gives a very simple result: the probability of transition per unit time per unit volume of a field ϕ in a potential $V(\phi)$ is $\Gamma/V = A\exp(-S_E)$, where S_E is the Euclidean action for ϕ evaluated along the "bounce" path of the field. The thin-wall approximation is realized when the ratio of the energy difference ϵ between the two vacuum states and the barrier height is much smaller than one. In this case S_E is simply[9]

$$S_E = \frac{27\pi^2 S_1^4}{2\epsilon^3}, \tag{13}$$

where $S_1 = \int_a^b d\phi\,[2V(\phi)]^{1/2}$, and $\phi = a, b$ are the two vacuum states.

In our case we have two fields, the dilaton σ (representing the inner-space dynamical variable) and the inflaton ψ, and, in general, several vacuum states. Assuming for ψ the potential Eq.(10), as required for the phase transitions occurring in the old, new, and extended inflationary models,[10] we have in fact three vacuum states (one of which is metastable), as previously shown. It is obvious that the tunnelling can occur along any path linking the vacuum states, but in Eq.(13) we need taking into account only the least-action path. In the same thin-wall approximation we can see that the only possible directions of tunnelling are from $\psi = 0$ to $\psi = \psi_T$ along $\sigma = 0$ with bounce action $S(\psi)$ (the desired tunnelling) or from $\sigma = 0$ to $\sigma = \infty$ along $\psi = 0$ with bounce action $S_E(\sigma)$ (the one to be avoided). We may then state that the inflaton tunnelling overrides the dilaton tunnelling if

$$S_E(\sigma) \gg S_E(\psi). \tag{14}$$

Let us finally start with the calculations. The complete potential for the two fields is Eq.(9) with $V(\psi)$ as in Eq.(10) [Fig. (4)]. For a small ϵ, the origin $\sigma = 0, \psi = 0$ is a (metastable) vacuum state, with $U(0,0) = \Lambda$. Let us call this vacuum state V_0. The other (true) vacuum states lie at ($\sigma = 0, \psi = \psi_T$), and (for any ψ) at $\sigma = +\infty$ (to be called V_1 and V_2, respectively). The Euclidean Action for the tunnelling $V_0 \to V_1$ is a well-known result, and in the thin-wall limit (small ϵ) it amounts to[11]

$$S_E(\psi) = \frac{\pi^2}{48\lambda\epsilon^3}. \tag{15}$$

The Euclidean Action in the σ direction can be evaluated in the thin-wall approximation if $\Lambda \ll U_M$, where U_M is the maximum of $U(\sigma, \psi = 0)$. To first order in $1/D$, i.e., when we may neglect the first term in Eq.(9), it is easy to see that the maximum is attained at $y_M = (1 - 2/D)$, with $U_M = 2\alpha/(De^2)$. Then the thin-wall condition is equivalent to $\Lambda/\alpha \ll 2/(D\, e^2)$ and is fulfilled for

$$D \gg 4\pi e^2 \epsilon \lambda \psi_0^4 G_N b_0^2. \tag{16}$$

We will comment later on this inequality.

The calculation of $S_E(\sigma)$ involves the integral

$$S_1 = \int_0^\infty d\sigma [2U(\sigma, \psi = 0)]^{1/2}, \tag{17}$$

that can be recast in the form (neglecting Λ/α)

$$S_1 = \sigma_0 \sqrt{2\alpha} \int_0^1 dy \left[\frac{2}{D+2} y^{2(D+1)} + y^{D-2} - \frac{D+4}{D+2} y^D \right]^{1/2}, \tag{18}$$

where we have defined $y = \exp(-\sigma/\sigma_0)$. Again, to first order in $1/D$, we have $S_1 = \sigma_0 F \sqrt{\alpha}/D$, where F is a geometric dimensionless factor of order unity, with a very mild dependence on D. When $D = 6$, for example, a numerical integration gives $F = 0.966$, while for $D = 20$ we get $F = 0.57$. Eq.(13) now reads

$$S_E(\sigma) = \frac{27}{2}\frac{\pi^2 F^4}{\Lambda^3}\left(\frac{D}{16\pi G_N b_0}\right)^4 . \tag{19}$$

The parameter Λ can be expanded in a power series in ϵ, and at lowest order is $\Lambda = \epsilon\lambda\psi_0^4/2$. Putting everything together, the inequality of Eq.(14) gives

$$\frac{D}{G_N b_0} \gg \beta\frac{m_\psi^3}{\lambda}, \qquad \beta = \frac{16\pi}{3F}, \tag{20}$$

where we introduced the ψ mass, $m_\psi^2 = \lambda\psi_0^2/2$. The constants appearing in Eq.(20) are all free parameters of the theory (except of course G_N), but they are in principle observable quantities.

Notice that although we assumed that ψ =const $= 0$, we do not expect the calculation to be changed much if ψ is slowly rolling as in new inflation. In particular, a successful new inflation, i.e., one which does not violate the constraint on the production of primordial fluctuations, either in the form of gravitational waves or scalar perturbations, must have very small m_ψ and λ. For example, in Planck units, it is often assumed $m_\psi \sim 10^{-6}$ and $\lambda \sim 10^{-12}$. The natural, yet unknown, value for b_0 is the Planck length, so that Eq.(20) is expected to be satisfied even for $D = 1$. Moreover, one can see that Eq.(16) is consistent with Eq.(20) when the same values as above are assumed, rendering inflation a good mechanism for having dimensional stability, at least in the thin-wall limit.

IV. CLASSICAL STABILITY

In Linde's chaotic inflation,[12] the field ψ need not have a potential of the form Eq.(10). Indeed, it is possible to have inflation for any ψ field evolving classically to zero starting from an initial value of a few Planck units (at least three Planck masses for producing 70 e-folds of inflation). From the modified potential for the dilaton field σ in Eq.(9) [coupled now with Eq.(12)—see Fig. (5)], one sees that, for large ψ, the potential barrier that makes $\sigma = 0$ a stable solution could disappear. Including ψ-dependent term, the total potential assumes the following form

$$U_C(\sigma,\psi) = \alpha\left\{\frac{2}{D+2}y^{2(D+2)} + [1+W(\psi)]y^D - \frac{D+4}{D+2}y^{D+2}\right\}, \tag{21}$$

with $W(\psi) = \lambda\psi^4/4\alpha$. It is not difficult to show that U_C has either two local extrema [a minimum and a maximum, as in Fig. (1)] or no local extrema at all, depending on the value of $W(\psi)$. This can be seen most easily by splitting the derivative $U' = \partial U_C/\partial y$ into two functions of y, $U_C'(y) = \alpha y^{D-1}[f_1(y) - f_2(y)]$, where $f_1 = 4y^{D+4}$ and $f_2 = (D+4)y^2 - D(1+W)$. It is clear that $f_2(y)$ crosses $f_1(y)$ at most two times (for $y > 0$), and that there must exist some W_*, and hence some ψ_*, for which f_2 is tangent to f_1. The value $\psi = \psi_*$ signals that the barrier has vanished, and that starting from ψ above this critical value the classical evolution will be toward $\sigma = +\infty$ (which we want to avoid). The critical value ψ_* can be determined exactly by solving the system in y and ψ

$$f_1' = f_2', \qquad f_1 = f_2. \tag{22}$$

From the first equation we learn that the barrier disappears when $y = y_* = 2^{-1/(D+2)}$, and from the second one that this happens when $W(\psi)$ has the value

$$W_* = y_*^2\left(1+\frac{2}{D}\right) - 1. \tag{23}$$

Then, we may state that the condition for the existence of a barrier between the compactified Universe ($\sigma = 0$) and the unfolded one ($\sigma = +\infty$) is, for large D,

$$V(\psi) = \frac{1}{4}\lambda\psi^4 \leq \frac{1}{4}\lambda\psi_*^4 = \frac{D}{8\pi G_N b_0^2}. \tag{24}$$

The last term in Eq.(24) is of order unity in Planck units if b_0 is close to the Planck length. In this case, the condition Eq.(24) is similar to the "quantum-boundary" constraint $V(\psi) < M_{Pl}^4$, and both inequalities may be satisfied assuming a very weakly coupled inflaton, as usually done in current inflationary scenarios.

Equation (24) has another, very interesting, implication. In Linde's chaotic inflation the initial value ψ_i of the field and its self-coupling constant λ are given a lower bound from the requirement of sufficient inflation ($\psi_i \geq 3M_{Pl}$), and of enough initial seed fluctuations to drive the subsequent large-scale structure formation[12,13] ($\lambda \geq 10^{-12}$). In this case, Eq.(24) implies that

$$b_0^2 \leq \frac{D}{8\pi G_N V(\psi_i)} \approx D \times 10^{10}\ l_{Pl}^2, \tag{25}$$

where l_{Pl} is the Planck length. If one considers that the experimental upper bound on b_0 is not better than $b_0 < 10^{17}\ l_{Pl}$, the purely theoretical speculations lead to an improvement of more than ten orders of magnitude. Notice that most theoretical bounds on the inner dimensions deal only with the rate of change of the inner radius, i.e., with $\dot{b}/b$ or with some compactification ratio b/b_0 (see, for example, upper bounds from nucleosynthesis[1] or microwave background anisotropy[14]). Here, in contrast, the very existence of a point (σ_*, ψ_*) at which the barrier disappears allows a direct upper bound on the absolute value of the present inner radius b_0. A similar constraint can be derived from Eq.(20), but there it rests on the hypothesis of thin-wall bubbles, and it is a less stringent bound.

Let us conclude this section observing that the shrinking of the barrier is a quite general feature, provided the self-coupling potential for ψ is monotonically growing, and that the shape of $U(\sigma, \psi = 0)$ for σ is as in Fig. (1).

V. CONCLUSIONS

In multidimensional theories, there exists an internal space of radius b_0 that is assumed to be very small and static. This configuration is made stable—both classically and semiclassically—by an appropriate potential. However, in any inflationary scenario this potential is modified so that an instability appears. Is, then, multidimensional cosmology incompatible with inflation?

We took in consideration old, new and extended inflation on one hand and chaotic inflation on the other. In the first case the problem turns out to be of a semiclassical nature: stability is preserved if the probability for the dilaton to tunnel through its potential is smaller than that for the inflaton to do the same under its own. A calculation of the transition rates in the thin-wall limit shows that this is actually the case; reasonable choices of the mass of the inflaton and of its self-coupling constant do not give rise to instability for any number of internal dimensions. In chaotic inflation, the problem is totally classical; for very large values of the inflaton field ψ, the potential barrier disappears and the internal space can grow without limit. Nevertheless, the initial conditions and the parameters of the model adjust themselves naturally in such a way as to allow for a successful inflation, and at the same time to meet the conditions for the barrier to exist. For both cases, the result is then that *the internal space remains stable during inflation.*

Extensions of Linde's model[15] predict that there are regions of the Universe in an eternal inflationary stage. This happens when, in one of the causally disconnected "miniuniverses," the scalar field ψ is initially greater than $\lambda^{-1/6} M_{Pl}$. In this case ψ grows larger and larger climbing the potential in Fig. (3) rather than rolling down to zero. However, the maximum value that ψ can reach is $\psi_{QB} \equiv \lambda^{-1/4} M_{Pl}$ at which its growth becomes suppressed.[15] In our multidimensional environment, this could imply that eventually ψ

becomes larger than ψ_* where the compactification breaks. Now, we see from Eq.(24) that $\psi_* \approx (D/b_0^2)^{1/4}\psi_{QB}$; then, depending on the values of b_0 and D, ψ_* lies either in the classical or in the quantum region. In the latter case ψ never reaches ψ_* where the barrier disappears, and we may conclude that in eternally inflating domains internal dimensions cannot be unfolded; in the former case, on the contrary, unfolding takes place, with the consequence that most part of the physical volume of the Universe lives in a multidimensional state. Of course one must have in mind that these considerations hold true only for compactification schemes of the kind we discussed in Section II, and that chaoticity allows in principle all kinds of dimensional dynamics in different miniuniverses.

One last result is worth of mention. The knowledge of the physical point in the (σ, ψ) plane at which the barrier disappears, allows a *direct bound on the present radius* of the internal space $b_0^2 \leq D \times 10^{10}\, l_{Pl}^2$, while, usually, only limits on the value of the ratio b/b_0 are given.

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FIGURE CAPTIONS

Fig. 1: Casimir potential $U_1(\sigma)$. The points with $\sigma = 0$ correspond to a stable compactification of the internal space when the vacuum energy vanishes.

Fig. 2: Inflationary potential with phase transition. Notice the ground state at $\psi = \psi_T$.

Fig. 3: Quartic potential for chaotic model. Inflation occurs when ψ rolls down to zero.

Fig. 4: Total potential of Eq.(9) in old, new or extended inflation. The Universe may tunnel from the origin toward two ground states. The most likely event is the one in which the internal space remains compactified.

Fig. 5: Potential of Eq.(9) in chaotic inflation. The barrier against evolution in the σ direction is seen to disappear for large ψ.

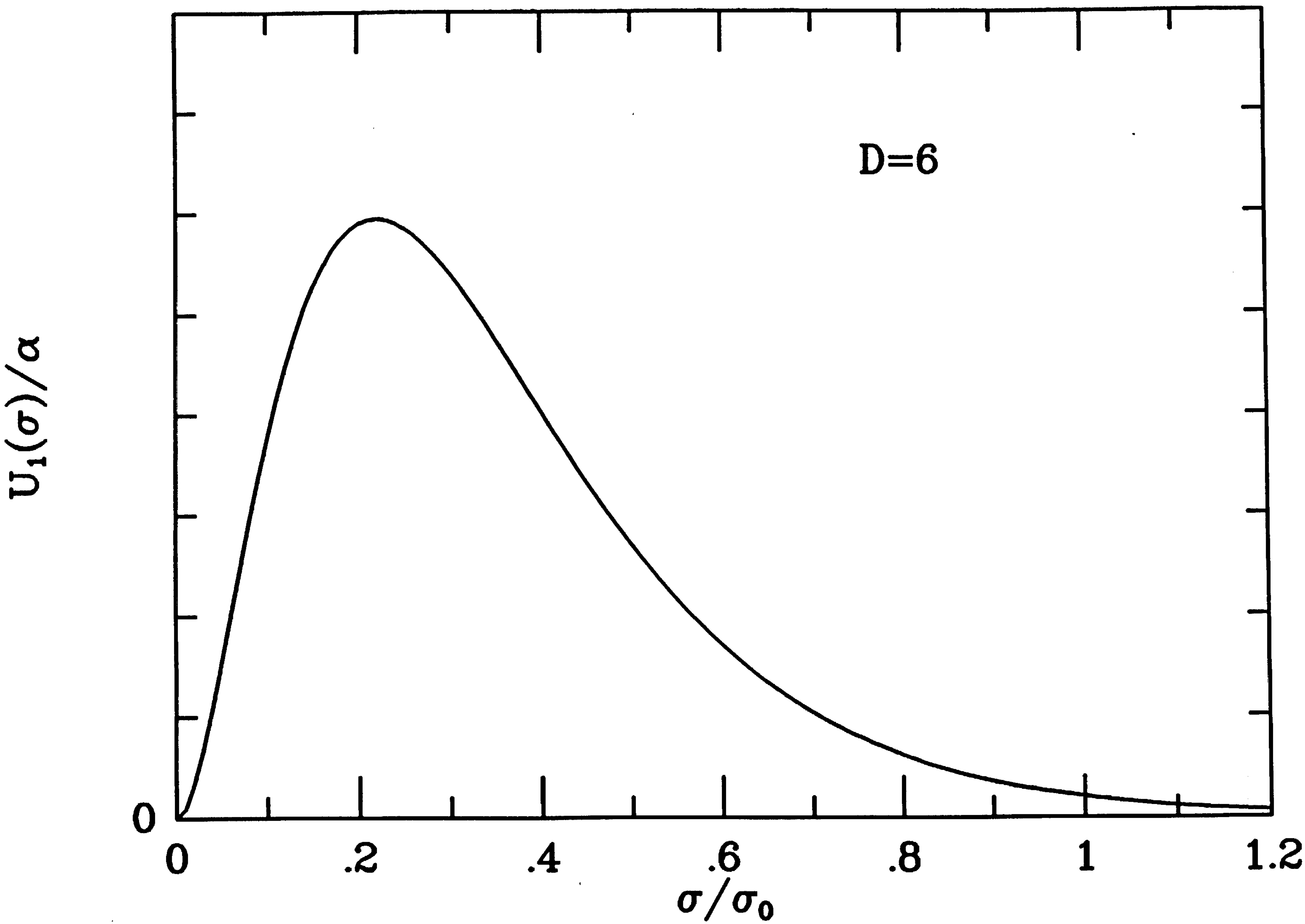


Fig. 1

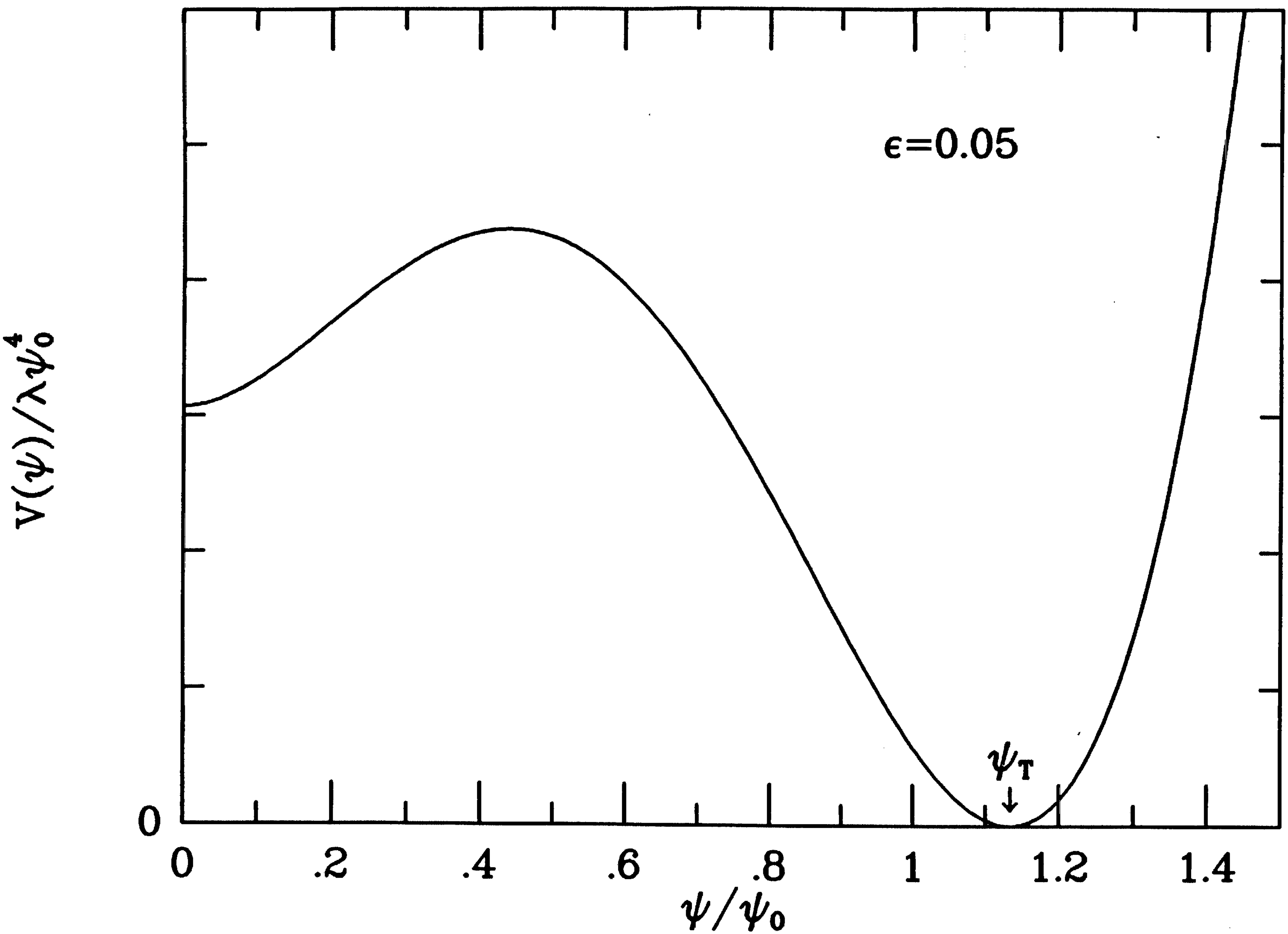


Fig 2.

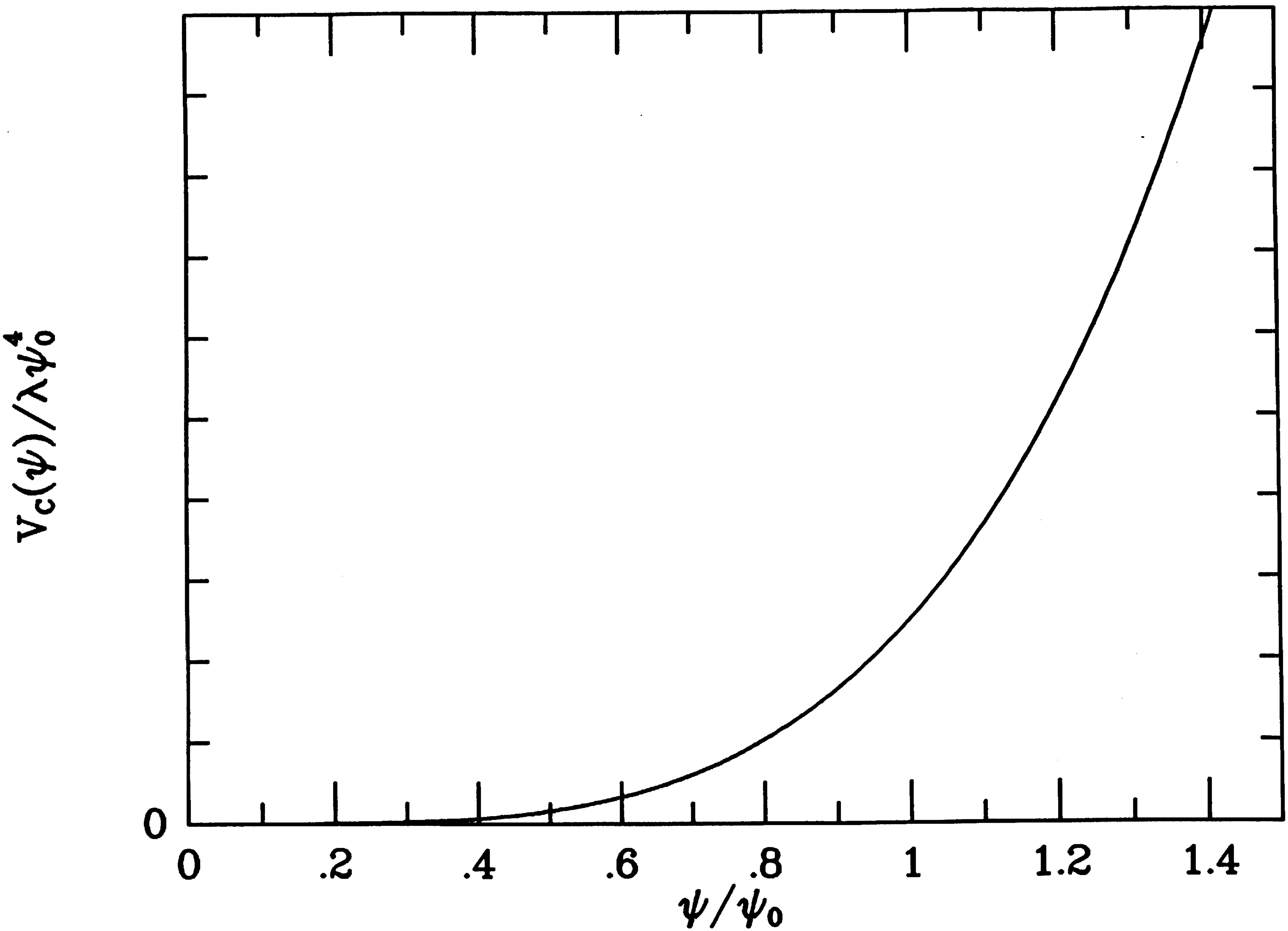
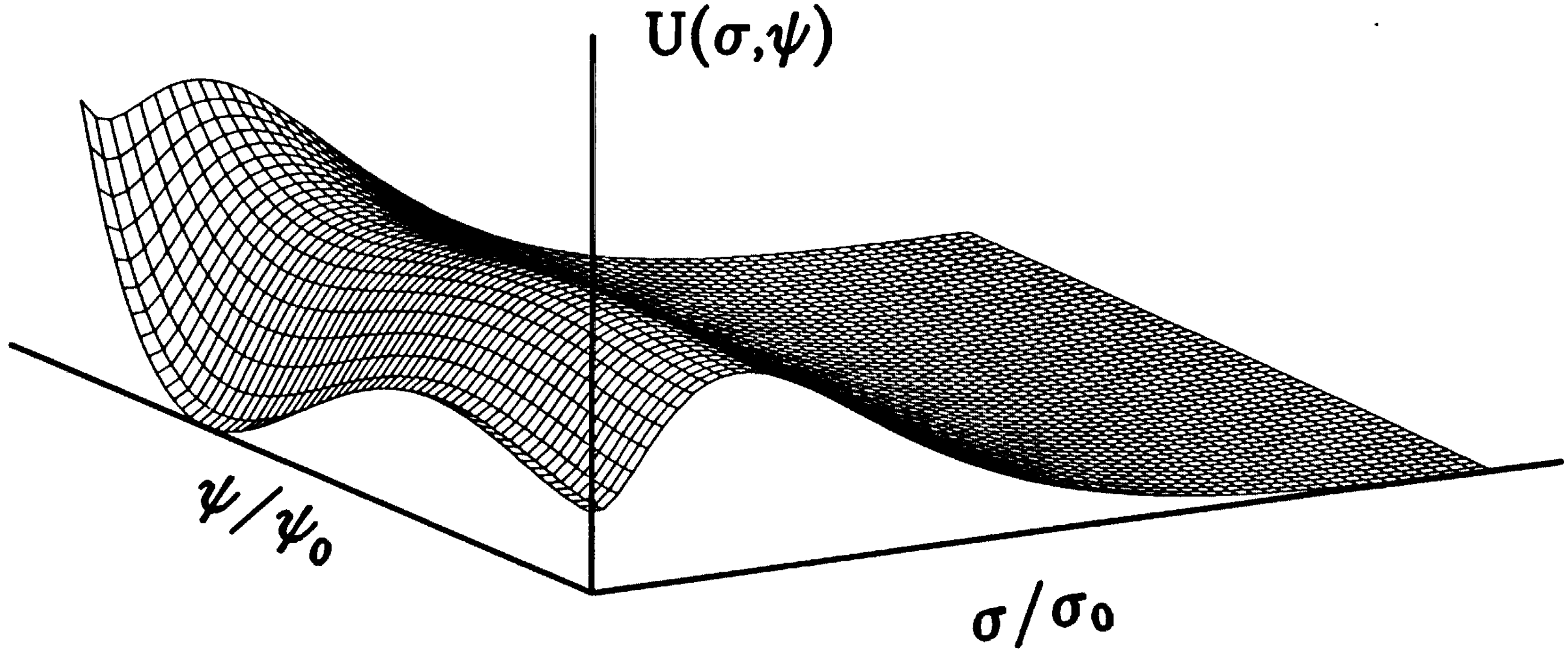


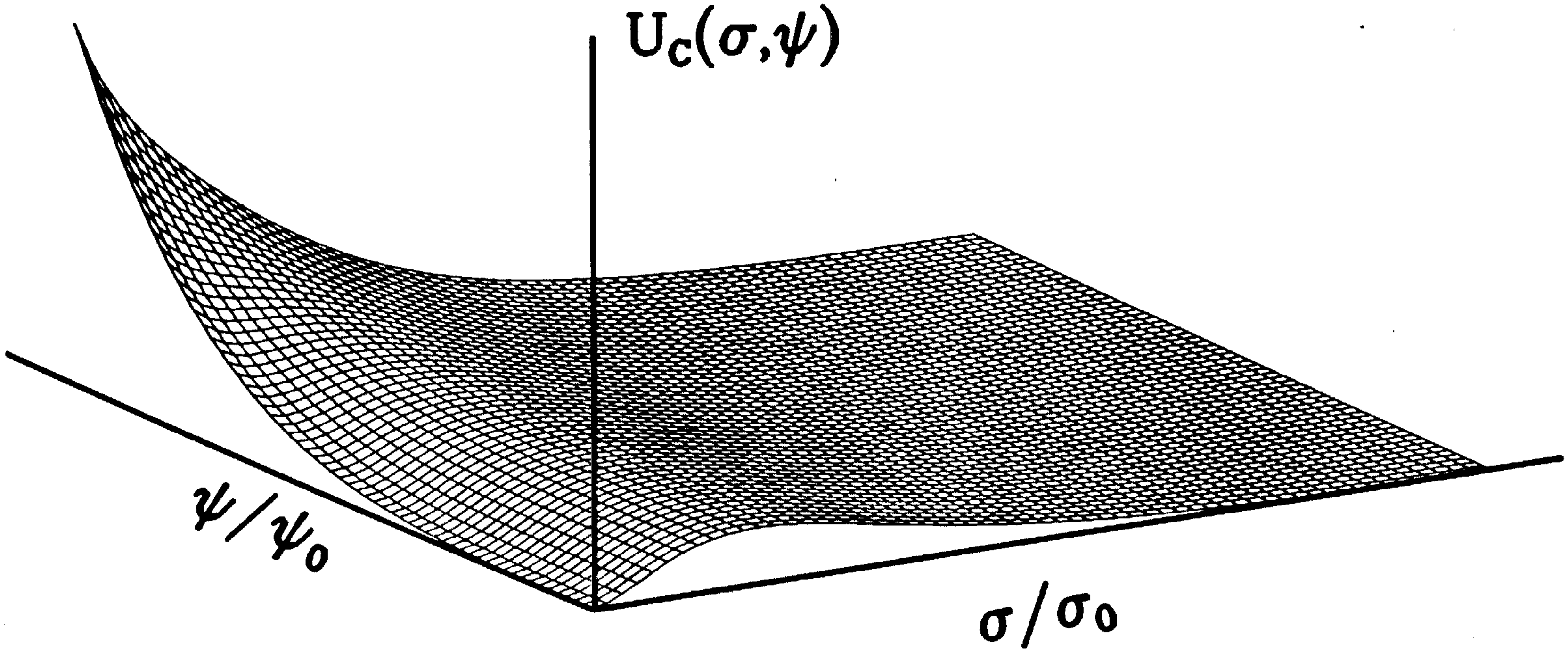
Fig. 3



Casimir (D=6)

$\alpha=1 \quad \lambda=1 \quad \epsilon=0.05$

Fig. 4



Casimir (D=6)

$\alpha=1 \quad \lambda=0.3$

Fig. 5